Lecture 8. **Frequency stability criteria**

Frequency criteria allow us to judge about system stability by its frequency characteristics. There are two frequency criteria we will consider later on: Mikhailov and Nyquist. Generalized frequency characteristic is obtained from transfer function $W(s)=\dfrac{Q_1(s)}{Q_2(s)}$ by substituting $s = j\omega$ i.e. $W(j\omega)=\dfrac{Q_1(j\omega)}{Q_2(j\omega)}$.

Thus, characteristic polynomial $Q_2(s)$or $Q_2(j\omega)$defines properties of dynamic system.

8.1 Mikhailov's stability criterion

In 1938 Russian scientist A. V. Mikhailov formulated the following stability criterion for linear systems of any order.

For stable closed-loop system of any order it is necessary and sufficient, that hodograph curve of vector $D(j\omega)$, which defines Mikhailov curve, meets the following requirements:

a) starts at positive real axis;

b) sequentially passes *"n"* quadrants;

c) have an angle of rotation $\varphi = n\dfrac{\pi}{2}$ (frequency changes from 0 to infinity);

d) goes to infinity in *n*th quadrant where *"n"* is the order of characteristic equation.

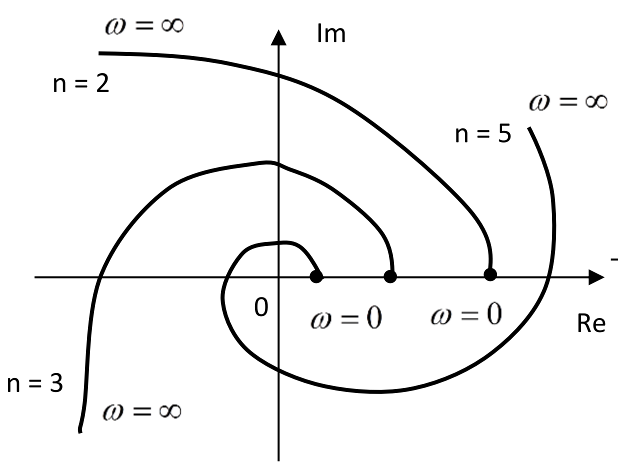

Fig. 3.5a. Hodograph curve of a stable ACS

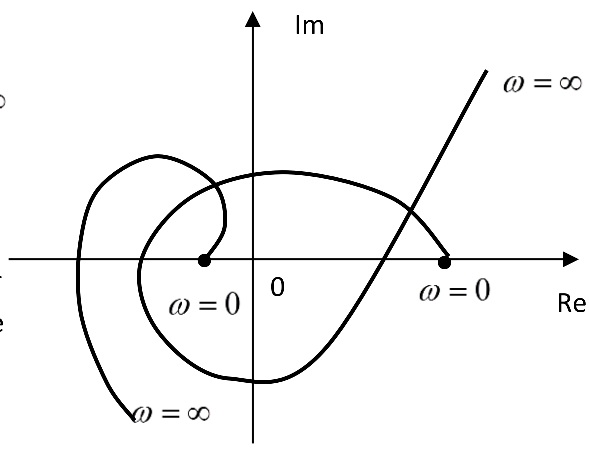

Fig. 3.5b. Hodograph curve of an unstable ACS

Above mentioned Mikhailov's curve *D(jω)* is a characteristic polynomial of a closed-loop system. That means if we have $W(j\omega)=\dfrac{Q_1(j\omega)}{Q_2(j\omega)}$ for a open-loop system, then for a closed-loop system it will be $W_{cl}(j\omega)=\dfrac{W(j\omega)}{1+W(j\omega)}=\dfrac{Q_1(j\omega)}{Q_1(j\omega)\ +Q_2(j\omega)}$ (taking into account the fact, that unit negative feedback is unit, i.e. $W_{Fb}(s)=1$).

Hence, $D(j\omega)=Q_1(j\omega)+Q_2(j\omega)$.

Definition of Mikhailov's curve is divided into real and imaginary parts:

$$D(j\omega)=X(\omega)+jY(j\omega)=\mathrm{Re}\,D(j\omega)+\mathrm{Im}\,D(j\omega).$$

From definition of such form we construct Mikhailov's hodograph curve changing ω from 0 to infinity. Obtained graph allows us to judge about stability of the closed-loop system. It is necessary to note, that since hodograph sequentially passes all *n* quadrants, roots of equations $X(\omega)=0$ and $Y(\omega)=0$ must alternate starting from real axis: $0=\omega_1<\omega_2<\omega_3<...<\omega_n$, where ω changes from 0 to infinity.

When we faced with algebraic stability criterions, the following three stability thresholds were found:

1) zero root or *aperiodic stability threshold* $a_n=0$;
2) $\Delta_{n-1}=0$ corresponding to *the oscillating stability threshold*;
3) *infinite root threshold*.

Existence of stability thresholds of all three types can be determined using Mikhailov's curve in the following way:

1) Stability threshold of the first type (zero root) lead to absence of free term in characteristic polynomial $a_n=0$, so the curve starts at the origin (Fig 3.6a).

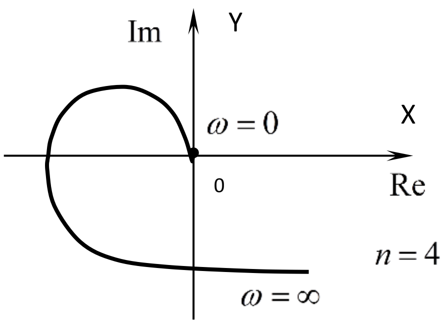

Fig. 3.6a. Stability threshold of the first type

2) Stability threshold of the second type (oscillating stability threshold) means that $D(j\omega_0)=0$, hence $X(\omega)=0$, $Y(\omega)=0$. This in turn means that point $\begin{cases}\omega=\omega_0\\ A=A_0\end{cases}$ of the curve lies at the origin (fig 3.6b).

3)

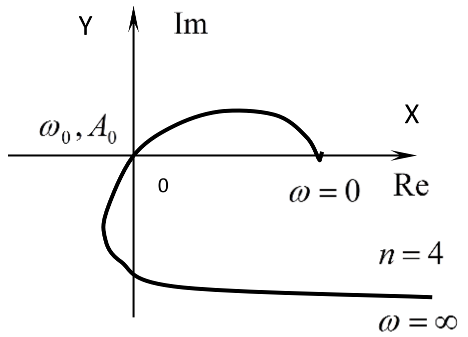

Fig. 3.6b. Stability threshold of the second type

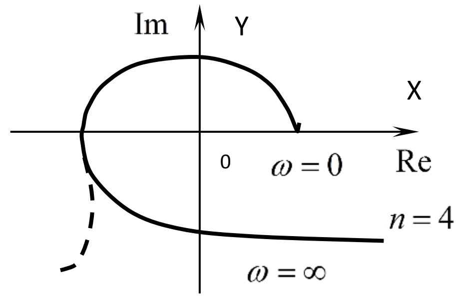

Fig. 3.6c. Stability threshold of the third type

Stability threshold of the third type (infinite root) is characterized by the flipped end of the curve in the III quarter (fig. 3.6c).

8.2 Nyquist's stability criterion

American scientist H. Nyquist formulated another type of criterion, the frequency stability criterion. In this criterion we can judge about a closed-loop system stability using gain-phase frequency characteristic (*GPhFC*) of the open-loop system. This is a big advantage of the method.

Criterion itself: *if the point with coordinates $(-1, j0)$ in complex plane is not contained within an interior of the GPhFC curve of an open-loop stable dynamic system, then the corresponding closed-loop system is stable* (fig. 3.3a - 3.7c).

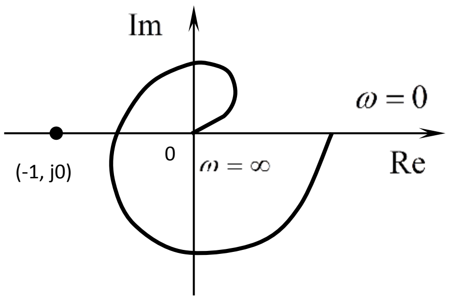

Fig. 3.7a. GPhFC of an open system

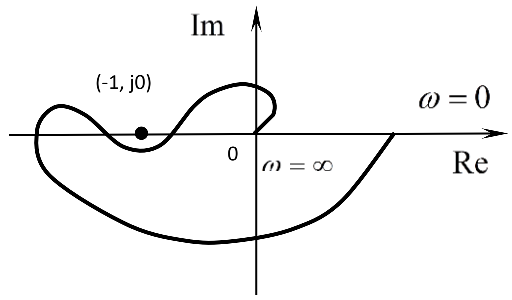

Fig. 3.7b. Hodograph of a stable system

This is a necessary and sufficient condition for stability of dynamic system with unit negative feedback.

So, now we have the criterion defined for systems with degenerative (unit negative) feedback. But where the point $(-1, j0)$ came from? As a rule, in mathematics "special" points are points of discontinuity at 0 or infinity, and here we have not usual $(-1, j0)$. A question to inquisitive reader: why coordinates of this special point are such ones?

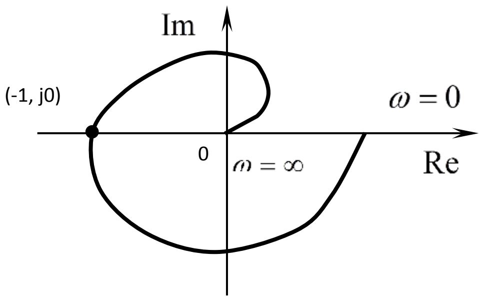

Fig. 3.7c. Hodograph of a board stability system

Normative Nyquist's criteria reflect the situation when feed backward connection is not unit (one) but has gain coefficient k_0.

Let *W(s)* be a transmission function of the open-loop system (fig 3.8).

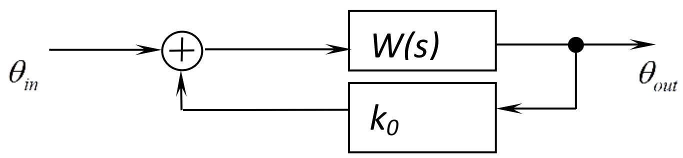

Fig. 3.8. The scheme of the system

Transmission function of the closed-loop system is $W_з(s)=\frac{W(s)}{1+k_0W(s)}$.

Normative Nyquist's criterion is characterized by definition of a special point $W_з(s)=\frac{\frac{1}{k_0}W(s)}{\frac{1}{k_0}+W(s)}$. This special point has coordinates $\left(-\frac{1}{k_0}, j0\right)$.

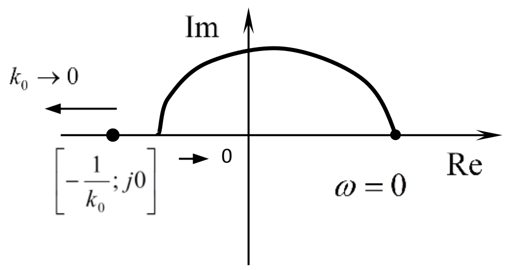

Fig. 3.9a. Open system GPhFC

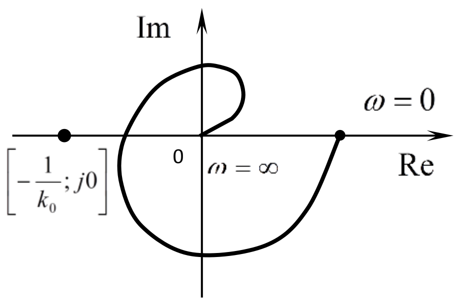

Fig. 3.9b. System hodograph

When feed backward connection coefficient k_0 decreases the system stability increases and vice versa.

Normative Nyquist's criterion interpretation is very useful. For systems of the 1st and 2nd order k_0 can be of any value. But for a system of the 3d order there is always critical amplification coefficient. All real systems can be described by differential equations. These are as a rule higher than third-order equations. As a result *all real systems have critical amplification coefficients.*

Nyquist's criterion was formulated for a stable open-loop system. But if an open-loop is unstable is it possible to make the system stable by means of feedback connection? Certainly it is possible.

In the case of unstable open-loop system for stability of closed-loop system it is necessary and enough to have hodograph (*GPhFC*) that embraces a special point *k* times. Here *"k"* is a number of roots in the right half-plane for open system .

The main conclusions of Nyquist's criteria interpretation:

– on GPhFC of an open system it is possible to judge about stability of an closed-loop system;

– for closed-loop system stability it is necessary to decrease feedback connection coefficient k_0.

8.3 Comparison of stability criteria

If ACS mathematical description was done (given) in terms "input-output", to check the system stability one can use criteria of Hurwitz, Mikhailov and Nyquist. If ACS mathematical description is done (given) in space states, to check system stability A.M. Lyapunov's theorems are used.

If the system order is $n \le 4$, it is advisable to use Hurwitz's criterion. If the system order is $n > 4$ Routh's criterion should be used. Mikhailov's stability criterion is advisable for complicated multilevel systems.

Nyquist's criterion fits for complicated systems with delay, for those described by means of analytical functions, and for the cases when links characteristics are given by experiments.

Lyapunov's theory is used when systems descriptions are given in space states. All of the described above stability criteria give possibility to conclude if the system is stable or not at the definite given parameters.

But sometimes it is necessary to solve the problem when one or two system parameters can change in definite limits and you should choose their optimal values that give the best dynamic factors of the ACS.

It turned out that there is a possibility to trace the impact of some parameters on the system stability. Some special methods were worked out and they will be considered below.